B.K. BIRLA CENTRE FOR EDUCATION

SARALA BIRLA GROUP OF SCHOOLS
A CBSE DAY-CUM-BOYS' RESIDENTIAL SCHOOL

PRE BOARD 3. (2025-26)

MATHEMATICS (041) SET-2

Class: XII A **Time: 3 hr**
Date: 06/01/26 **Max Marks: 80**
Admission no: **Roll no:**

General Instructions: -

Read the following instructions very carefully and strictly follow them:

1. This Question paper contains 38 questions. All questions are compulsory.
2. This Question paper is divided into five Sections - A, B, C, D and E.
3. In Section A, Questions number 1 to 18 are multiple choice questions (MCQs) with only one correct option and Questions number 19 and 20 are Assertion-Reason based questions of 1 mark each.
4. In Section B, Questions number 21 to 25 are Very Short Answer (VSA)-type questions, carrying 2 marks each.
5. In Section C, Questions number 26 to 31 are Short Answer (SA)-type questions, carrying 3 marks each.
6. In Section D, Questions number 32 to 35 are Long Answer (LA)-type questions, carrying 5 marks each.
7. In Section E, Questions number 36 to 38 are Case study-based questions, carrying 4 marks each.
8. There is no overall choice. However, an internal choice has been provided in 2 questions in Section B, 3 questions in Section C, 2 questions in Section D and one subpart each in 2 questions of Section E.
9. Use of calculator is not allowed

SECTION-A

(This section comprises of multiple-choice questions (MCQs) of 1 mark each.)

1. Which of the following statements is not true about equivalence classes $A_i (i = 1, 2, 3, \dots n)$ formed by an equivalence relation **R** defined on set **A**. 1

(A) $\bigcup_{i=1}^{n} A_i = A$
(B) $A_i \cap A_j \neq \emptyset, i \neq j$
(C) $x \in A_i$ and $x \in A_j \Rightarrow A_i = A_j$
(D) All elements of A_i are related to each other, for all i

2. If $P(a, b, 0)$ lies on the line $\frac{x+1}{2} = \frac{y+2}{3} = \frac{z+3}{4}$, then (a, b) is 1

(A) $(1, 2)$ (B) $\left(\frac{1}{2}, \frac{1}{3}\right)$ (C) $\left(\frac{1}{2}, \frac{1}{4}\right)$ (D) (0,0)

3. The domain of the function $\cos^{-1}(2x - 1)$ is 1

(A) $[0, 1]$ (B) $[-1, 1]$ (C) $(0, 1)$ (D) $[0, 1)$

4. If inverse of matrix $\begin{bmatrix} 7 & -3 & -3 \\ -1 & 1 & 0 \\ -1 & 0 & 1 \end{bmatrix}$ is the matrix $\begin{bmatrix} 1 & 3 & 3 \\ 1 & \lambda & 3 \\ 1 & 3 & 4 \end{bmatrix}$, then the value of λ is 1

(A) -4 (B) 1 (C) 3 (D) 4

5. The corner points of the feasible region of a Linear Programming Problem are $(0, 2)$, $(3, 0), (6, 0), (6, 8)$ and $(0, 5)$. If $Z = ax + by; (a, b > 0)$ be the objective function, and maximum value of Z is obtained at $(0, 2)$ and $(3, 0)$, then the relation between a and b is given by 1

(A) $a = b$ (B) $3a = 2b$ (C) $a = 3b$ (D) $b = 6a$

6. If $\begin{bmatrix} x+y & 2 \\ 5 & xy \end{bmatrix} = \begin{bmatrix} 6 & 2 \\ 5 & 8 \end{bmatrix}$, then the value of $\frac{24}{x} + \frac{24}{y}$ is 1

(A) 7 (B) 6 (C) 8 (D) 18

7. If the area of triangle with vertices $(2, -6), (5, 4)$ and $(k, 4)$ is 35 sq. unit then k is 1

(A) -1.5 (B) 8.5 (C) $-12, 2$ (D) $12, -2$

8. Given a square matrix A of order 3×3, such that $|A| = 12$, then the value of $|A \text{ adj } A|$ is 1

(A) 1728 (B) 144 (C) 72 (D) 12

9. If the matrix $A = \begin{bmatrix} 0 & 5 & -7 \\ a & 0 & 3 \\ b & -3 & c \end{bmatrix}$ is a skew symmetric matrix, then the value of $2a - 3b + c$ is 1

(A) -11 (B) -31 (C) 1 (D) 31

10. Find $\frac{d}{dx}[\cos(\log x + e^x)]$ at $x = 1$ 1

(A) $-\sin e$ (B) $\sin e$ (C) $-(1 + e)\sin e$ (D) $(1 + e)\sin e$

11. A cylindrical tank of radius 10 cm is being filled with sugar at the rate of $100\pi cm^3/s$. Find the rate, at which the height of the sugar inside the tank is increasing. 1

(A) $0.1cm/s$ (B) $0.5cm/s$ (C) $1cm/s$ (D) $1.1cm/s$

12. Evaluate $\int \frac{e^{-x}}{16+9e^{-2x}} dx$ 1

(A) $-\frac{1}{12}\tan^{-1}\left(\frac{3e^{-x}}{4}\right) + c$ (B) $\frac{16}{9}\tan^{-1}(e^{-x}) + c$

(C) $\tan^{-1}\left(\frac{e^{-x}}{4}\right) + c$ (D) $-\frac{1}{3}\tan^{-1}\left(\frac{e^{-x}}{4}\right) + c$

13. Evaluate $\int \frac{1}{\sin^2 x \cos^2 x} dx$ 1

(A) $\tan x + \cot x + c$ (B) $\tan x - \cot x + c$

(C) $\cot x - \tan x + c$ (D) $-\tan x . \cot x + c$

14. If p and q are the order and degree of the differential equation $\left(\frac{d^2y}{dx^2}\right)^2 + 3\frac{dy}{dx} + \frac{d^3y}{dx^3} = 4$ respectively, then the value of $2p - 3q$ is 1

(A) 7 (B) -7 (C) 3 (D) -3

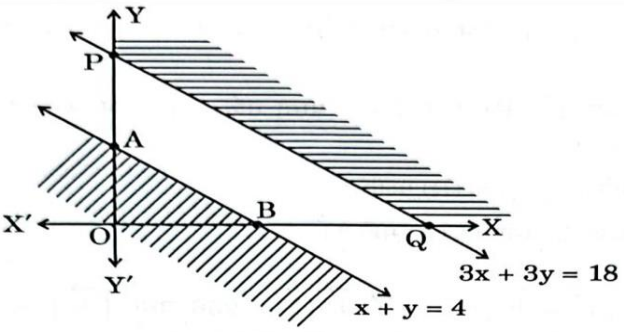
15. If the function $f(x) = \begin{cases} 3x - 8, & x \leq 5 \\ 2k, & x > 5 \end{cases}$ is continuous, then the value of k is 1

(A) $\frac{2}{7}$ (B) $\frac{7}{2}$ (C) $\frac{3}{7}$ (D) $\frac{4}{7}$

16. In a Linear Programming problem, the objective function $Z = 2x + 5y$ is to be maximized under the following constraints: 1

$$x + y \leq 4, 3x + 3y \geq 18, x, y \geq 0$$

Study the graph and select the correct option.



The solution of the given LPP:

(A) lies in the shaded unbounded region
(B) lies in ΔAOB
(C) does not exist
(D) lies in the combined region of ΔAOB and unbounded shaded region

17. The position vectors of points P and Q are $\vec{p}$ and $\vec{q}$ respectively. The point R divides line segment PQ in the ratio 3 : 1 and S is the midpoint of line segment PR. The position vector of S is 1

(A) $\frac{\vec{p}+3\vec{q}}{4}$ (B) $\frac{\vec{p}+3\vec{q}}{8}$ (C) $\frac{5\vec{p}+3\vec{q}}{4}$ (D) $\frac{5\vec{p}+3\vec{q}}{8}$

18. The angle which the line $\frac{x}{1} = \frac{y}{-1} = \frac{z}{0}$ makes with the positive direction of Y-axis is 1

(A) $\frac{3\pi}{4}$ (B) $\frac{7\pi}{4}$ (C) $\frac{5\pi}{4}$ (D) $\frac{5\pi}{6}$

ASSERTION-REASON BASED QUESTIONS

Direction: Question numbers 19 and 20 are Assertion-Reason based questions carrying 1 mark each. Two statements are given, one labelled Assertion (A) and the other labelled Reason (R). Select the correct answer from the options (A), (B), (C) and (D) as given below.

(A) Both (A) and (R) are true and (R) is the correct explanation of (A).
(B) Both (A) and (R) are true but (R) is not the correct explanation of (A).
(C) (A) is true but (R) is false.
(D) (A) is false but (R) is true.

19. **Assertion (A):** All trigonometric functions are invertible over their entire domains. 1

Reason (R): The inverse function $\tan^{-1} x$ exists for all real numbers $x \in R$

20. **Assertion (A):** If vectors $\vec{a} = 3\hat{\imath} + \hat{\jmath} + 4\hat{k}$ and $\vec{b} = \hat{\imath} - \hat{\jmath} + \hat{k}$ denote the adjacent sides of a parallelogram, then the area of parallelogram is $\sqrt{42}$ 1

Reason (R): If $\vec{a}$ and $\vec{b}$ represent the adjacent sides of a parallelogram, then its area is given by $|\vec{a} \times \vec{b}|$.

SECTION B

(This section comprises of 5 very short answer (VSA) type questions of 2 marks each.)

21. Find the principal value of $\cos^{-1}(\cos\frac{13\pi}{6})$ 2

OR

Find the value of $\tan^{-1}(\sqrt{3}) - \sec^{-1}(-2)$

22. If $y = (\sin^{-1} x)^2$, then show that $(1 - x^2)\frac{d^2y}{dx^2} - x\frac{dy}{dx} - 2 = 0$ 2

23. If $y = (\tan x)^x$, then find $\frac{dy}{dx}$ 2

24. Evaluate $\int_0^{2\pi} |\sin x|\, dx$ 2

OR

Find the area of the region enclosed by the curve $y^2 = x, x = 3$ and the $x - axis$ in the first quadrant

25. If $\vec{a} = \hat{\imath} + \hat{\jmath} + \hat{k},\ \vec{a}.\vec{b} = 1$ and $\vec{a} \times \vec{b} = \hat{\jmath} - \hat{k}$, then find $|\vec{b}|$ 2

SECTION C

(This section comprises of 6 short answer (SA) type questions of 3 marks each.)

26. In the Linear Programming Problem (LPP), find the point/points giving the maximum value for $Z = 5x + 10y$ 3

Subject to constraints:

$$x + 2y \leq 120$$
$$x + y \geq 60$$
$$x - 2y \geq 0$$
$$x, y \geq 0$$

27. If $x = a(\cos t + t\sin t), y = a(\sin t - t\cos t)$ then find $\frac{d^2y}{dx^2}$ 3

OR

If $\sqrt{1 - x^2} + \sqrt{1 - y^2} = a(x - y)$, prove that $\frac{dy}{dx} = \sqrt{\frac{1-y^2}{1-x^2}}$

28. Find the intervals in which the function f given by 3

$f(x) = \sin x + \cos x, 0 \leq x \leq 2\pi$ is increasing or decreasing.

29. Evaluate $\int \frac{x^{-1}}{(\log x)^2 - 5\log x + 4} dx$ 3

30. Find the area of the region bounded by the curves $x^2 = y, y = x + 2$ and $x - axis$, using integration. 3

OR

Find the area of the ellipse $\frac{x^2}{16} + \frac{y^2}{9} = 1$ using integration.

31. Write the vector equations of the following lines and hence find the shortest distance between them: 3

$$\frac{x+1}{2} = \frac{y+1}{-6} = \frac{z+1}{1} \text{ and } \frac{x-3}{1} = \frac{y-5}{-2} = \frac{z-7}{1}$$

OR

Find the equation of a line in vector and cartesian form which passes through the point $(1, 2, -4)$ and is perpendicular to the line $\frac{x-8}{3} = \frac{y+19}{-16} = \frac{z-10}{7}$ and $\vec{r} = (15\hat{\imath} + 29\hat{\jmath} + 5\hat{k}) + \mu(3\hat{\imath} + 8\hat{\jmath} - 5\hat{k})$.

SECTION - D

(This section comprises of 4 long answer (LA) type questions of 5 marks each)

32. If $A = \begin{bmatrix} 2 & -3 & 5 \\ 3 & 2 & -4 \\ 1 & 1 & -2 \end{bmatrix}$, then find A^{-1}. 5

Hence, solve the system of linear equations:

$$2x - 3y + 5z = 11$$
$$3x + 2y - 4z = -5$$
$$x + y - 2z = -3$$

33. Solve the differential equation 5

$\left(x\sin^2\left(\frac{y}{x}\right) - y\right)dx + xdy = 0$ given that $y = \frac{\pi}{4}$ when $x = 1$

OR

Solve the differential equation $\frac{dy}{dx} - 3y\cot x = \sin 2x$ given that $y = 2$ when $x = \frac{\pi}{2}$

34. Evaluate: 5

$$\int_0^{\pi} \frac{x\tan x}{\sec x + \tan x}dx$$

OR

Evaluate: $\int_0^5 (|x-1| + |x-2| + |x-5|)dx$

35. Find the image of the point $(-1, 5, 2)$ in the line $\frac{2x-4}{2} = \frac{y}{2} = \frac{2-z}{3}$. 5

Also find the length of the line segment joining the points. (i.e. given point and the image point).

SECTION- E

(This section comprises of 3 case-study/passage-based questions of 4 marks each with subparts. The first two case study questions have three subparts (I), (II), (III) of marks 1, 1, 2 respectively. The third case study question has two subparts of 2 marks each)

36. A window is in the form of a rectangle surmounted by an equilateral triangle on its length. Let the rectangular part have length and breadth $\boldsymbol{x}$ and $\boldsymbol{y}$ metres respectively. 4

Based on the given information, answer the following questions:

(i) If the perimeter of the window is 12 m, find the relation between $\boldsymbol{x}$ and $\boldsymbol{y}$. (1)

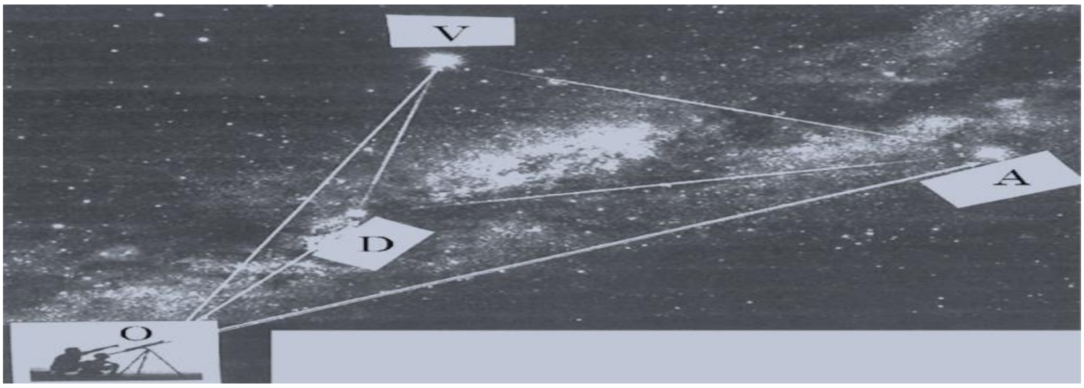
(ii) Using the expression obtained in (i), write an expression for the area of the window as a function of $\boldsymbol{x}$ only. (1)

(iii) (A) Find the dimensions of the rectangle that will allow maximum light through the window. (use expression obtained in (ii)) (2)

OR

(iii) (B) If it is given that the area of the window is 50 m², find an expression for its perimeter in terms of $\boldsymbol{x}$

37. An instructor at the astronomical centre shows three among the brightest stars in a particular constellation. Assume that the telescope is located at O(0, 0, 0) and the three stars have their locations at the points D, A and V having positions vectors $2\hat{\imath} + 3\hat{\jmath} + 4\hat{k}, 7\hat{\imath} + 5\hat{\jmath} + 8\hat{k}$ and $-3\hat{\imath} + 7\hat{\jmath} + 11\hat{k}$ respectively. 4



Based on the above information, answer the following questions:

(i) Find the measure of acute angle $\angle VDA$ (2)

(ii) What is the projection of vector $\overrightarrow{DV}$ on vector $\overrightarrow{DA}$. (2)

38. 4

Mathematics subject teacher wants to assess the learning of his students of the concept of "functions" taught to them. He writes the following five relations, each defined from set A = {1, 2, 3} to set B= {a, b, c, d}

$R_1 = \{(1, a), (1, b), (1, c), (1, d)\}$

$R_2 = \{(1, a), (2, b), (3, c), (3, d)\}$

$R_3 = \{(1, b), (2, b), (3, c)\}$

$R_4 = \{(1, a), (2, b), (3, c)\}$

$R_5 = \{(1, a), (2, a), (3, a)\}$

The students are asked to answer the following questions about the above relations:

(i) Identify the relations which are functions from A to B (1)

(ii) Identify the relations which are injective (1)

(iii)(A) Check the injectivity and surjectivity of the function $g: \boldsymbol{N} \to \boldsymbol{N}$ given by $g(x) = x^2$ (2)

OR

(iii) (B) Let P= $\{1, 2, 3, 4, \ldots 30\}$ be the set of roll numbers of 30 students of a class. R be the relation defined by teacher to plan seating arrangement of students in pairs, where $R = \{(x, y): x, y$ are Roll Numbers of students such that $y = 3x\}$.
Write R in roster form. Is the relation R reflexive, symmetric and transitive? Justify your answer.